(3) 7.2 *The Exponential Function (B)*

Example 16 Let $f(x) = (2 + 7^x)^5$ Find $f^{\backslash}(x)$

Solution

$f^{\backslash}(x) = 5(2 + 7^x)^4 \cdot 7^x \ln 7$

Example 17 Let $f(x) = \dfrac{2^x - 1}{3^x + 2}$ Find $f^{\backslash}(x)$

Solution

$f^{\backslash}(x) = \dfrac{2^x \ln 2 \cdot (3^x + 2) - 3^x \ln 3 \cdot (2^x - 1)}{(3^x + 2)^2}$

Example 18 Let $f(x) = \dfrac{e^x + 3}{e^x + 1}$ Find $f^{\backslash}(x)$

Solution

$f^{\backslash}(x) = \dfrac{e^x(e^x + 1) - e^x(e^x + 3)}{(e^x + 1)^2}$

$= \dfrac{e^x(e^x + 1 - e^x - 3)}{(e^x + 1)^2} = \dfrac{-2e^x}{(e^x + 1)^2}$

Integral:

$\displaystyle\int a^x \, dx = \dfrac{1}{\ln a} a^x + c$	$\displaystyle\int 7^x \, dx = \dfrac{1}{\ln 7} 7^x + c$
$\displaystyle\int e^x \, dx = e^x + c$	
$\displaystyle\int \dfrac{1}{x} \, dx = \ln|x| + c$	

Example 19 find $\displaystyle\int 2^{-x}6^x\,dx$ 24 March   2008 A

Solution

$$2^{-x}\,6^x = \left(\frac{1}{2}\right)^x 6^x = \left(\frac{6}{2}\right)^x = 3^x$$

$$I = \int 2^{-x}\,6^x\,dx = \int 3^x\,dx = 3^x \cdot \frac{1}{\ln 3} + c$$

Example 20 find $\displaystyle\int 2^{-x}e^{x+3}\,dx$ 19 March 2006 A

Solution

$$2^{-x}e^{x+3} = \left(\frac{1}{2}\right)^x e^x \cdot e^3 = e^3 \left(\frac{e}{2}\right)^x$$

$$I = \int 2^{-x}\,e^{x+3}\,dx = e^3 \int \left(\frac{e}{2}\right)^x dx = e^3 \left(\frac{e}{2}\right)^x \frac{1}{\ln\frac{e}{2}} + c$$

Example 21 Evaluate $\displaystyle\int (3^{-x} - 2^{x-1})^2\,dx$ 25 April   2008

Solution

$$(3^{-x} - 2^{x-1})^2 = 3^{-2x} - 2\cdot 3^{-x}\cdot 2^{x-1} + 2^{2x-2}$$

$$2\cdot 3^{-x}\cdot 2^{x-1} = \left(\frac{1}{3}\right)^x \cdot 2^x = \left(\frac{2}{3}\right)^x$$

$$I = \int (3^{-x}\cdot 2^{x-1})^2\,dx = \int \left(3^{-2x} - \left(\frac{2}{3}\right)^x + 2^{2x-2}\right)dx$$

$$= 3^{-2x}\cdot\frac{1}{\ln 3}\cdot\frac{-1}{2} - \left(\frac{2}{3}\right)^x\frac{1}{\ln\left(\frac{2}{3}\right)} + 2^{2x-2}\cdot\frac{1}{\ln 2}\cdot\frac{1}{2} + c$$

Example 22 find $\displaystyle\int \frac{(2^{-x}-2^x)^2}{2^x}\,dx$

Solution

$$\left(\frac{2^{-x}-2^x}{2^x}\right)^2 = \frac{2^{-2x}-2+2^{2x}}{2^x} = 2^{-3x} - 2^{1-x} + 2^x$$

$$I = \int \frac{(2^{-x}-2^x)^2}{2^x}\,dx = \int (2^{-3x} - 2^{1-x} + 2^x)\,dx$$

$$= 2^{-3x}\cdot\frac{1}{\ln 2}\cdot\frac{-1}{3} - 2^{1-x}\cdot\frac{1}{\ln 2}\cdot(-1) + 2^x\cdot\frac{1}{\ln 2} + c$$

Example 23 find $\displaystyle\int \frac{e^x}{3 + e^{x+2}}\,dx$

Solution

$u = 3 + e^{x+2}$ $\qquad du = e^{x+2}\,dx$ $\qquad du = e^2 \cdot e^x\,dx$ $\qquad \dfrac{1}{e^2}\,du = e^x\,dx$

$$I = \int \frac{e^x}{3 + e^{x+2}}\,dx = \frac{1}{e^2}\int \frac{1}{u}\,du = \frac{1}{e^2}\ln|u| + c = \frac{1}{e^2}\ln(3 + e^{x+2}) + c$$

Example 24 find $\displaystyle\int \frac{e^3}{1 + e^{-x}}\,dx$

Solution

$$I = \int \frac{e^3}{1 + e^{-x}}\,dx = e^3\int \frac{e^x}{e^x + 1}\,dx$$

$$u = e^x + 1 \qquad\qquad du = e^x\,dx$$

$$I = e^3\int \frac{1}{u}\,du = e^3\ln|u| + c = e^3\ln(1 + e^x) + c$$

Example 25 find $\displaystyle\int \frac{\cos(2^{-x})}{2^x}\,dx$

Solution

$u = 2^{-x}$ $\qquad du = 2^{-x} \cdot \ln 2\,(-1)\,dx$ $\qquad \dfrac{-1}{\ln 2}\,du = \dfrac{1}{2^x}\,dx$

$$I = \frac{-1}{\ln 2}\int \cos u\,du = \frac{-1}{\ln 2}\cdot \sin u + c = \frac{-1}{\ln 2}\cdot \sin(2^{-x}) + c$$

Example 26 find $\displaystyle\int \frac{5^{\cot x}}{\sin^2 x}\,dx$ 15 July 2003 A

Solution

$u = \cot x$ $\qquad du = -\csc^2 x\,dx$ $\qquad -du = \dfrac{1}{\sin^2 x}\,dx$

$$I = \int \frac{5^{\cot x}}{\sin^2 x}\,dx = -\int (5)^u\,du = -(5)^u \cdot \frac{1}{\ln 5} + c = -5^{\cot x}\cdot \frac{1}{\ln 5} + c$$

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Example 27   find $\int x^2 4^{-x^3}\, dx.$ 10 March 1999

Solution

$u = -x^3$ $du = -3x^2\, dx$ $\dfrac{-1}{3} du = x^2\, dx$

$$I = \frac{-1}{3}\int (4)^u\, du = \frac{-1}{3}\cdot (4)^u \cdot \frac{1}{\ln 4} + c = \frac{-1}{3\ln 4}\cdot 4^{-x^3}$$

Example 28   find $\int \dfrac{dx}{4^{-x}\sqrt{4^x - 4}}$ 2 March 1993

Solution

$u = 4^x - 4$ $du = 4^x \cdot \ln 4\, dx$ $\dfrac{1}{\ln 4} du = \dfrac{1}{4^{-x}} dx$

$$I = \int \frac{dx}{4^{-x}\sqrt{4^x - 4}} = \frac{1}{\ln 4}\int \frac{1}{\sqrt{u}}\, du = \frac{2}{\ln 4}\sqrt{u} + c = \frac{2}{2\ln 2}\sqrt{4^x - 4} + c = \frac{1}{\ln 2}\sqrt{4^x - 4} + c$$

Homework

1 Evaluate the following integral $\int 3^{-x}9^x\,dx$

2 Evaluate the following integral $\int 3^x e^{x+1}\,dx$

3 Evaluate the following integral $\int (2^x + 3^{2x})^2\,dx$ 23 Nov. 2007 A

4 Evaluate the following integral $\int \dfrac{(2^{x+1} - 2^{3x})^2}{2^x}\,dx$ 5 October 1996

5 Evaluate the following integral $\int \dfrac{(e^{-x} + e^{2x})^2}{e^{3x}}\,dx$ 7 July 1997

6 Evaluate the following integral $\int \dfrac{e^x}{1 + e^{x-2}}\,dx$

7 Evaluate the following integral $\int \dfrac{e^{2x+3}}{1 + e^{2x}}\,dx$

8 Evaluate the following integral $\int \dfrac{e^{2x}\,dx}{\sqrt{10 - e^{2x}}}$ 3 November 1994

9 Evaluate the following integral $\int \dfrac{\cos(5^x)}{5^{-x}}\,dx$

10 Evaluate the following integral $\displaystyle\int_0^{\frac{\pi}{4}} \dfrac{2^{\tan x}}{\cos^2 x}\,dx$ 11 October 1999

11 Evaluate the following integral $\int x^2 4^{-x^3}\,dx.$ 10 March 1999